Economic sanctions as deterrents and constraints

Online appendix

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Online appendix

Strategy profile 1

Let $u_s(s_0, a_0^*) > u_s(s_i, a_i^*) \, \forall \, i \neq j$ and $u_s(s_0, a_0^*) > u_s(s_j, a_j^*)$. Solve for the conditions under which the following strategy profile is a sub-game perfect Nash equilibrium.

> Sender: If in any previous round the sender played s_0, the target played $a > \underline{a_j}$ and the sender has not subsequently played s_j, then play s_j. If the target has never played $a > \underline{a_j}$ in response to s_0, or if the target has played $a > \underline{a_j}$ in response to s_0 and the sender has subsequently played s_j, then play s_0.
>
> Target: If the target has previously played $a > \underline{a_j}$ in response to s_0, the sender has not subsequently played s_j, and in the current round the sender plays s_0, then play a_0^*. If the target has never played $a > \underline{a_j}$ in response to s_0 or if the target has played $a > \underline{a_j}$ in response to s_0 and the sender subsequently played s_j, and the sender plays s_0 in the current round then play $\underline{a_0}$. If the sender plays s_i, play a_i^*. If the sender plays s_j, play a_j^*.

Deviations by the sender

Let the history be that the target has never played $a > \underline{a_j}$ in response to s_0, or if the target has played $a > \underline{a_j}$ in response to s_0 then the sender has subsequently played s_j. If the sender follows its strategy and plays s_0 it will receive $u_s(s_0, \underline{a_j}) + \frac{\delta_s u_s(s_0, \underline{a_j})}{1-\delta_s}$. If it deviates and plays s_i it will receive $u_s(s_i, a_i^*) + \frac{\delta_s u_s(s_0, \underline{a_i})}{1-\delta_s}$, and if it deviates and plays s_j it will receive $u_s(s_j, a_j^*) + \frac{\delta_s u_s(s_0, \underline{a_j})}{1-\delta_s}$. Since $u_s(s_o, \underline{a_j}) > u_s(s_i, a_i^*) \, \forall \, i \neq j$ and $u_s(s_o, \underline{a_j}) > u_s(s_j, a_j^*)$, the sender will not deviate.

Let the history be that in a previous round the target has played $a > \underline{a_j}$ in response to s_0 and the sender has not subsequently played s_j. If the sender follows its strategy and plays s_j it will receive $u_s(s_j, a_j^*) + \delta_s u_s(s_0, \underline{a_j}) + \frac{\delta_s^2 u_s(s_0, \underline{a_j})}{1-\delta_s}$. If it deviates to s_0 it will receive

$u_s(s_0, a_0^*) + \delta_s u_s(s_j, a_j^*) + \frac{\delta_s^2 u_s(s_0, \underline{a_j})}{1-\delta_s}$. If it deviates to s_i it will receive $u_s(s_i, a_i^*) + \delta_s u_s(s_j, a_j^*) + \frac{\delta_s^2 u_s(s_0, \underline{a_j})}{1-\delta_s}$. Therefore, for the sender not to deviate

$$u_s(s_j, a_j^*) + \delta_s u_s(s_0, \underline{a_j}) \geq u_s(s_i, a_i^*) + \delta_s u_s(s_j, a_j^*) \; \forall \; i \neq j,$$

$$u_s(s_j, a_j^*) + \delta_s u_s(s_0, \underline{a_j}) \geq u_s(s_0, a_0^*) + \delta_s u_s(s_j, a_j^*),$$

must hold. These are Conditions (1) and (2).

Deviations by the target

Let the history be that the target previously played $a > \underline{a_j}$ in response to s_0, the sender has not subsequently played s_j, and the sender plays s_0 in this round. If the target follows its strategy and plays a_0^* it will receive $u_t(s_0, a_0^*) + \delta_t u_t(s_j, a_j^*) + \frac{\delta_t^2 u_t(s_0, \underline{a_j})}{1-\delta_t}$. If the target deviates and plays $a \neq a_0^*$ it will receive $u_t(s_0, a \neq a_0^*) + \delta_t u_t(s_j, a_j^*) + \frac{\delta_t^2 u_t(s_0, \underline{a_j})}{1-\delta_t}$. Since a_0^* is the utility maximizing value of a given s_0, $u_t(s_0, a_0^*) > u_t(s_0, a \neq a_0^*)$ so the target will not deviate.

Let the history be that the target previously played $a > \underline{a_j}$ in response to s_0, the sender has not subsequently played s_j, and the sender plays s_j in this round. If the target follows its strategy and plays a_j^*, then it will receive $u_t(s_j, a_j^*) + \frac{\delta_t u_t(s_0, \underline{a_j})}{1-\delta_t}$. If the target deviates and plays $a \neq a_j^*$ it will receive $u_t(s_j, a \neq a_j^*) + \frac{\delta_t u_t(s_0, \underline{a_j})}{1-\delta_t}$. Since a_j^* is the utility maximizing value of a given s_j, $u_t(s_j, a_j^*) > u_t(s_j, a \neq a_j^*)$ so the target will not deviate.

Let the history be that the target previously played $a > \underline{a_j}$ in response to s_0, the sender has not subsequently played s_j, and the sender plays s_i in this round. If the target follows its strategy and plays a_i^*, then it will receive $u_t(s_i, a_i^*) + \delta_t u_t(s_j, a_j^*) + \frac{\delta_t^2 u_t(s_0, \underline{a_j})}{1-\delta_t}$. If the target deviates and plays $a \neq a_i^*$ it will receive $u_t(s_i, a \neq a_i^*) + \delta_t u_t(s_j, a_j^*) + \frac{\delta_t^2 u_t(s_0, \underline{a_j})}{1-\delta_t}$. Since a_i^* is the utility maximizing value of a given s_i, $u_t(s_i, a_i^*) > u_t(s_i, a \neq a_i^*)$ so the target will not deviate.

Let the history be that the target has never played $a > \underline{a_j}$ in response to s_0, or if the target has played $a > \underline{a_j}$ in response to s_0 the sender has subsequently played s_j, and that the sender has played s_0 in this round. If the target follows its strategy and plays $\underline{a_j}$ then it will receive $u_t(s_0, \underline{a_j}) + \delta_t u_t(s_0, \underline{a_j}) + \frac{\delta^2 u_t(s_0,\underline{a_j})}{1-\delta_t}$. If it deviates to $a < \underline{a_j}$ it will receive $u_t(s_0, a < \underline{a_j}) + \delta_t u_t(s_0, \underline{a_j}) + \frac{\delta^2 u_t(s_0,\underline{a_j})}{1-\delta_t}$. Since $u_t(s_0, \underline{a_j}) > u_t(s_0, a < \underline{a_j})$, the target will not deviate to $a < \underline{a_j}$. If the target deviates to $a > \underline{a_j}$, its utility maximizing level of a will be a_0^*. If the target plays a_0^* it will receive $u_t(s_0, a_0^*) + \delta_t u_t(s_j, a_j^*) + \frac{\delta^2 u_t(s_0,\underline{a_j})}{1-\delta_t}$. Therefore, for the target not to deviate

$$u_t(s_0, \underline{a_j}) + \delta_t u_t(s_0, \underline{a_j}) \geq u_t(s_0, a_0^*) + \delta_t(s_j, a_j^*)$$

must hold. This is Condition (3)

Let the history be that the target has never played $a > \underline{a_j}$ in response to s_0, or if the target has played $a > \underline{a_j}$ in response to s_0 the sender has subsequently played s_j, and that the sender has played s_i in this round. If the target follows its strategy and plays a_i^* it will receive $u_t(s_i, a_i^*) + \frac{\delta_t u_t(s_0,\underline{a_j})}{1-\delta_t}$. If the target deviates to $a \neq a_i^*$ it will receive $u_t(s_i, a \neq a_i^*) + \frac{\delta_t u_t(s_0,\underline{a_j})}{1-\delta_t}$. Since a_i^* is the utility maximizing value of a given s_i, $u_t(s_i, a_i^*) > u_t(s_i, a \neq a_i^*)$ so the target will not deviate.

Let the history be that the target has never played $a > \underline{a_j}$ in response to s_0, or if the target has played $a > \underline{a_j}$ in response to s_0 the sender has subsequently played s_j, and that the sender has played s_j in this round. If the target follows its strategy and plays a_j^* it will receive $u_t(s_j, a_j^*) + \frac{\delta_t u_t(s_0,\underline{a_j})}{1-\delta_t}$. If the target deviates to $a \neq a_j^*$ it will receive $u_t(s_j, a \neq a_j^*) + \frac{\delta_t u_t(s_0,\underline{a_j})}{1-\delta_t}$. Since a_j^* is the utility maximizing value of a given s_j, $u_t(s_j, a_j^*) > u_t(s_j, a \neq a_j^*)$ so the target will not deviate.

Therefore, provided that Conditions (1), (2) and (3) hold, the strategy profile is an SPNE. QED.

Strategy profile 2

Let $u_s(s_k, a_k^*) \geq u_s(s_0, a_0^*)$ and $u_s(s_k, a_k^*) \geq u_s(s_i, a_i^*) \; \forall \; i \neq k$. Solve for the conditions under which the following strategy profile is a sub-game perfect Nash equilibrium.

Sender: If in any previous period the sender played s_0 and then the target defected by playing $a > \underline{a_k}$, play s_k forever. If the target has never defected by playing $a > \underline{a_k}$ when the sender played s_0, play s_0.

Target: If the target defected in any previous round by playing $a > \underline{a_k}$ in response to the sender playing s_0, and the sender defects on punishing in the current round by playing s_0, play a_0^*. If $s_0, a > \underline{a_k}$ was not played in any previous round, and the sender plays s_0 in the current round, play $\underline{a_k}$. If the sender plays s_i play a_i^*, and if the sender plays s_k play s_k^*.

Deviations by the sender

Let the history be that in a previous period the target deviated and played $a > \underline{a_k}$ when the sender played s_0. If the sender follows its strategy and plays s_k it gets $u_s(s_k, a_k^*) + \frac{\delta_s u_s(s_k, a_k^*)}{1-\delta_s}$. If the sender deviates and plays s_0 it gets $u_s(s_0, a_0^*) + \frac{\delta_s u_s(s_k, a_k^*)}{1-\delta_s}$. If the sender deviates and plays s_i it gets $u_s(s_i, a_i^*) + \frac{\delta_s u_s(s_k, a_k^*)}{1-\delta_s}$. Since $u_s(s_k, a_k^*) \geq u_s(s_0, a_0^*)$ and $u_s(s_k, a_k^*) \geq u_s(s_i, a_i^*) \, \forall \, i \neq k$ the sender has no incentive to deviate.

Let the history be that the target has never deviated and played $a > \underline{a_k}$ when the sender played s_0. If the sender follows its strategy and plays s_0 it gets $u_s(s_0, \underline{a_k}) + \frac{\delta_s u_s(s_0, \underline{a_k})}{1-\delta_s}$. If the sender deviates and plays s_i it gets $u_s(s_i, a_i^*) + \frac{\delta_s u_s(s_0, \underline{a_k})}{1-\delta_s}$. If the sender deviates and plays s_k it gets $u_s(s_k, a_k^*) + \frac{\delta_s u_s(s_0, \underline{a_k})}{1-\delta_s}$. To prevent the sender from deviating and imposing sanctions when its strategy does not call for them

$$u_s(s_0, \underline{a_k}) \geq u_s(s_i, a_i^*) \, \forall \, i \neq k,$$

$$u_s(s_0, \underline{a_k}) \geq u_s(s_k, a_k^*)$$

must hold. These are Conditions (4) and (5).

Deviations by the target

Let the history be that in a previous period the sender played s_0, the target played $a > \underline{a_k}$ and in this period the sender has played s_0. If the target follows its strategy and plays a_0^* it gets $u_t(s_0, a_0^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. If the target deviates and plays $a \neq a_0^*$ it will receive $u_t(s_0, a \neq a_0^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. Since a_0^* is the utility maximizing value of a given s_0, $u_t(s_0, a_0^*) > u_t(s_0, a \neq a_0^*)$ so the target will not deviate.

Let the history be that in a previous period the sender played s_0, the target played $a > \underline{a_k}$ and in this period the sender has played s_i. If the target follows its strategy it gets $u_t(s_i, a_i^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. If the target deviates and plays $a \neq a_i^*$ it will receive $u_t(s_i, a \neq a_i^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. Since a_i^* is the utility maximizing value of a given s_i, $u_t(s_i, a_i^*) > u_t(s_i, a \neq a_i^*)$ so the target will not deviate.

Let the history be that in a previous period the sender played s_0, the target played $a > \underline{a_k}$ and in this period the sender has played s_k. If the target follows its strategy it gets $u_t(s_k, a_k^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. If the target deviates and plays $a \neq a_k^*$ it will receive $u_t(s_k, a \neq a_k^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. Since a_k^* is the utility maximizing value of a given s_k, $u_t(s_k, a_k^*) > u_t(s_k, a \neq a_k^*)$ so the target will not deviate.

Let the history be that the target has never played $a > \underline{a_k}$ in response to the sender playing s_0 and in this period the sender has played s_0. If the target follows its strategy and plays $\underline{a_k}$ it gets $u_t(s_0, \underline{a_k}) + \frac{\delta_t u_t(s_0, \underline{a_k})}{1-\delta_t}$. If the target deviates to $a < \underline{a_k}$ it will receive $u_t(s_0, a < \underline{a_k}) + \frac{\delta_t u_t(s_0, \underline{a_k})}{1-\delta_t}$. Since $u_t(s_0, \underline{a_k}) > u_t(s_0, a < \underline{a_k})$ this is not a profitable deviation. If the target deviates to $a > \underline{a_k}$ it will receive $u_t(s_0, a > \underline{a_k}) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. The utility maximizing level of a given s_0 is a_0^*. Therefore for the target not to deviate

$$\frac{u_t(s_0, \underline{a_k})}{1-\delta_t} \geq u_t(s_0, a_0^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$$

must hold. This is Condition (6).

Let the history be that the target has never played $a > \underline{a_k}$ in response to the sender playing s_0 and in this period the sender has played s_i. If the target follows its strategy it gets $u_t(s_i, a_i^*) + \frac{\delta_t u_t(s_0, \underline{a_k})}{1-\delta_t}$. If the target deviates and plays $a \neq a_i^*$ it will receive $u_t(s_i, a \neq a_i^*) + \frac{\delta_t u_t(s_0, \underline{a_k})}{1-\delta_t}$. Since a_i^* is the utility maximizing value of a given s_i, $u_t(s_i, a_i^*) > u_t(s_i, a \neq a_i^*)$ so the target will not deviate.

Let the history be that the target has never played $a > \underline{a_k}$ in response to the sender playing s_0 and in this period the sender has played s_k. If the target follows its strategy it gets $u_t(s_k, a_k^*) + \frac{\delta_t u_t(s_0, \underline{a_k})}{1-\delta_t}$. If the target deviates and plays $a \neq a_k^*$ it will receive $u_t(s_k, a \neq a_k^*) + \frac{\delta_t u_t(s_0, \underline{a_k})}{1-\delta_t}$. Since a_k^* is the utility maximizing value of a given s_k, $u_t(s_k, a_k^*) > u_t(s_k, a \neq a_k^*)$ so the target will not deviate.

Therefore, provided that Conditions (4), (5) and (6) hold, the strategy profile is an SPNE. QED.

Strategy profile 3

Let $u_s(s_k, a_k^*) \geq u_s(s_o, a_0^*)$ and $u_s(s_k, a_k^*) \geq u_s(s_i, a_i^*) \forall i \neq k$. Show that the following strategy profile is a sub-game perfect Nash equilibrium.

Sender: Play s_k.

Target: If the sender plays s_k, play a_k^*. If the sender plays s_0, play a_0^*. If the sender plays s_i, play a_i^*.

Deviations by the sender

If the sender plays s_k it receives $u_s(s_k, a_k^*) + \frac{\delta_s u_s(s_k, a_k^*)}{1-\delta_s}$. If the sender plays s_0 it receives $u_s(s_0, a_0^*) + \frac{\delta_s u_s(s_k, a_k^*)}{1-\delta_s}$. If the sender plays s_i it receives $u_s(s_i, a_i^*) + \frac{\delta_s u_s(s_k, a_k^*)}{1-\delta_s}$. Since $u_s(s_k, a_k^*) \geq u_s(s_0, a_0^*)$ and $u_s(s_k, a_k^*) \geq u_s(s_i, a_i^*) \; \forall \; i \neq k$, the sender will not deviate.

Deviations by the target

If the sender plays s_k and the target plays a_k^* it will receive $u_t(s_k, a_k^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. If it plays $a \neq a_k^*$ it will receive $u_t(s_k, a \neq a_k^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. Since a_k^* is the utility maximizing level of a

given s_k, $u_t(s_k, a_k^*) > u_t(s_k, a \neq a_k^*)$, so the target does not have a profitable deviation.

If the sender plays s_0 and the target plays a_0^* it will receive $u_t(s_0, a_0^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. If it plays $a \neq a_0^*$ it will receive $u_t(s_0, a \neq a_0^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. Since a_0^* is the utility maximizing level of a given s_0, $u_t(s_k, a_0^*) > u_t(s_k, a \neq a_0^*)$, so the target does not have a profitable deviation.

If the sender plays s_i and the target plays a_i^* it will receive $u_t(s_i, a_i^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. If it plays $a \neq a_i^*$ it will receive $u_t(s_i, a \neq a_i^*) + \frac{\delta_t u_t(s_k, a_k^*)}{1-\delta_t}$. Since a_i^* is the utility maximizing level of a given s_i, $u_t(s_i, a_i^*) > u_t(s_i, a \neq a_i^*)$, so the target does not have a profitable deviation.

Therefore the strategy profile is an SPNE. QED.